

METHOD FOR TWO-DIMENSIONAL REPRESENTATION, INTERPOLATION AND COMPRESSION OF DATA

[0001] Priority is claimed to German Patent Application No. DE 102 48 543.7, the subject matter of which is hereby incorporated by reference herein.

BACKGROUND

[0002] The present invention relates to a method for multi-dimensional representation, interpolation and compression of data. More specifically, it relates to the use of two-dimensional sampling functions that are defined over the complex numbers to efficiently represent, interpolate, smooth, or compress data.

[0003] In the field of data processing, it is often required to compress data in order to speed up transmission or to improve processing. Therefore, there is a number of compression algorithms that use interpolation methods. Here, mention should be made of known compression methods for images, such as JPEG.

SUMMARY OF THE INVENTION

[0004] An object of the present invention is to provide a method which allows efficient compression of information, using interpolation algorithms that meet special requirements.

[0005] The invention provides a method for representation and/or interpolation and/or compression of automatically processable data. The method includes using a two-dimensional interpolation formula $s(z)$ based on a sampling function $a(z)$. The Cauchy integral theorem and, possibly, the residue theorem are applicable for the interpolation formula.

[0006] The methods used here have the special feature that holomorphic or

meromorphic functions are used. A detailed description of these technical terms can be gathered, for example, from A. Hurwitz, Vorlesungen über allgemeine Funktionentheorie und Elliptische Funktionen [Lectures on General Theory of Functions and Elliptic Functions], fifth edition, Springer, Berlin Heidelberg New York, 2000, which is hereby incorporated by reference herein. If a function in an open subset is differentiable with respect to a complex variable, then the function on this subset is referred to as "holomorphic". A function is meromorphic if a function has at most a finite number of poles in a limited region in the complex numbers and if, outside of these poles, the function is holomorphic in the complex numbers.

[0007] Applicable to such functions are theorems from the theory of functions which can be used for two-dimensional signal processing and signal compression. This holds especially true for a two-dimensional sampling function that is based on the Sinus Lemniscatus (see Gauss, Mathematisches Tagebuch [Mathematical Diary] 1796-1814, Harri Deutsch, 1985, which is hereby incorporated by reference herein). The behavior of such a two-dimensional sampling function is exceptionally suitable for sampling and interpolation purposes.

[0008] The method is suitable for interpolation, smoothing and compression of data. The special feature is the applicability of function-theoretical methods, which is made possible by satisfaction of the Cauchy-Riemann conditions. Also suitable is a Sinus Lemniscatus based interpolation formula which is based on the function $sl(z)/z$, as described later, and which yields identical interpolation functions in the two dimensions (x and y axes).

[0009] Using the Cauchy integral theorems and the residue theorem, the new representation can be used for compressing data.

BRIEF DESCRIPTION OF THE DRAWINGS

[0010] The invention is elaborated upon below based on exemplary embodiments with reference to the drawings, in which:

Figure 1 shows a two-dimensional sampling function $a(z) = sl \left(\frac{\bar{z}}{\pi z} \right) / \left(\frac{\bar{z}}{\pi z} \right)$;

Figure 2 shows a closed curve C with sampling points within C ;

Figure 3 shows the real part of $s(z)$ on C ;

Figure 4 shows the imaginary part of $s(z)$ on C ;

DETAILED DESCRIPTION

[0011] A point $P=(x, y)$, which is given by the real values x and y in Cartesian coordinates, is represented as a complex number $z=x+i*y$, with i being the square root of -1 .

[0012] The case under discussion is that of equidistant sampling; that is, the values z_j , for which sampled values exists, lie on the intersection points of a grid. For the sake of simplicity, normalization to unit distance is done, and a subset of the so-called "Gaussian integers" is taken as the set of sampling points. The Gaussian integers are complex numbers whose real and imaginary parts are integers. The values of two-dimensional functions $s(z)$ at sampled values z_j are given by s_j . Values s_j mostly belong to the complex numbers or to the real numbers, or to finite approximations of these numbers, such as are used for information processing in technical equipment.

[0013] An essential feature of the functions $s(z)$ discussed in the present invention is

that they satisfy the Cauchy-Riemann differential equations, except at possibly existing pole positions; that is, they are holomorphic or meromorphic.

[0014] Satisfaction of these equations is the fundamental condition for the applicability of the so-called Cauchy integral theorems or of the residue theorem. According to this theorem, the values of $s(z)$ within a closed curve C can be calculated using the values of the function $s(z)$ that are located on the boundary of the area bounded by C . This opens a number of new possibilities for the representation, interpolation and compression of data.

[0015] Suitable functions for use as a function $s(z)$ are especially those having zeros at least over the set of sampling points z_j , except at point $z=0$. These functions include, for example, suitably selected polynomials.

[0016] The following is a summary of the advantageous properties of $a(z)$:

1. It holds that $a(z)=1$ or another suitable constant.
2. It holds that $a(z_j)=0$, with $z_j \neq 0$ being a Gaussian integer.
3. The function is holomorphic, except at possibly occurring pole positions.

[0017] The function $s(z)$ is now expressed as:

$$s(z) = \sum s_j \cdot a(z-z_j).$$

[0018] Suitable for $a(z)$ are numerous functions having zeros at least at the sampling points. Moreover, in order to obtain functions that behave in a suitable manner during interpolation and approximation, it is convenient if the subset of the Gaussian integers that are zeros of the function $a(z)$ extend beyond sampled values z_j at least to curve C .

[0019] A function formed by the Sinus Lemniscatus is especially suitable for practical applications. The Sinus Lemniscatus is a function introduced by Gauss (see Gauss, Mathematisches Tagebuch [Mathematical Diary] 1796-1814, Harri Deutsch, 1985) which is similar to the known sine function and which can be represented using the well-known Jacobian elliptic functions. It holds, for example, that $sl(z) = (1/\sqrt{2}) \cdot sd(\sqrt{2}z, 1/\sqrt{2})$, where sd can be expressed, for example, using the standard functions sn and dn as follows: $sd(z,k) = sn(z,k) / dn(z,k)$ (see, for example, E.T. Whittaker, G.N. Watson, A course of modern analysis, fourth edition, Cambridge, reprinted 1969, page 524, which is hereby incorporated by reference herein).

[0020] Using the Sinus Lemniscatus, in short $sl(z)$, extending the well-known one-dimensional sampling function $\sin(\pi x)/(\pi x)$ (see, for example, C. Shannon, Communication in the presence of noise, Proceedings Institute of Radio Engineers, Vol. 36, 1948, pp.10-21, which is hereby incorporated by reference herein), it is possible to form a two-dimensional function

$$a(z) = \frac{sl(\pi z)}{\pi z}$$

[0021] The role of π in the one-dimensional sampling function is assumed by the value

In Figure 1, the function $a(z)$ is shown for real $z=x$ in the range $-2 < x < 2$.

- [0022] A remarkable feature of function $a(z)$ is that it holds that $a(iz)=a(z)$; that is, that the function is 90 degree rotationally invariant in the complex plane. In particular, the function yields the same values for purely imaginary values as for real values, and therefore has the property that the sampling function is identical in both dimensions, which is very convenient for two-dimensional interpolation. This is a property which is not found in the classical sampling function because in the case of purely imaginary arguments, the classical sine function can be represented by the exponential function \sinh .
- [0023] The very good behavior of $sl(z)/z$ is particularly advantageous for sampling and interpolation. This is also shown by the so-called "Fourier transform".
- [0024] As can be seen from the Fourier transform, the function is very close to the low-pass behavior of the $\sin(x)/x$ in terms of the frequency components.
- [0025] According to the well-known Cauchy integral theorems, holomorphic functions within a closed curve can be determined by the values on this curve. A corresponding typical scenario is depicted in Figure 2.
- [0026] Sampling points z_j are represented by small circles. Knowing the function $s(z)$ at sampling points z_j , that is, $s(z_j)=s_j$, it is possible to calculate function $s(z)$ according to $s(z) = \sum s_j \cdot a(z-z_j)$ also at the points of curve C . The points on the curve are referred to by variable τ .
- [0027] If sampling function $a(z)$ has no poles within C , then, according to the Cauchy integral theorem, it is true for values z within C that:

$$s(z) = \frac{1}{2\pi i} \oint_C \frac{s(\tau)d\tau}{\tau - z}$$

the curve being passed through in mathematically positive direction.

[0028] If function $a(z)$ has poles, then it is possible to extend the formula

$$s(z) = \frac{1}{2\pi i} \oint_c \frac{s(\tau)d\tau}{\tau - z} \text{ by including the corresponding residues using the}$$

well-known residue theorem.

[0029] In this context, it is essential that the points within C can be determined by the points τ on curve C . Thus, we have a universal method for storing two-dimensional data, but also general data known as values s_j , using values on curve C .

[0030] In the case of meromorphic functions, possibly, residues have to be included as well. In this context, the path length, for example, can be used for parameterization.

[0031] Depending on the redundancy of data s_j , it is possible to make do with less data on the curve. This results in the compression effect.

EXAMPLE

[0032] The present invention will now be described by way of an example.

[0033] Discussed are the points of Figure 2; the values of function $s(z) = si(\pi x/5) \cdot si(\pi y/5)$ being taken as the sampled values $s_j(z_j)$, where $z=x+iy$ and $si(x)=sin(x)/x$. The complex sampling points z_j are the $17 \cdot 13 = 221$ Gaussian integers within curve C .

[0034] Curve C is composed of four straight-line pieces C_1, C_2, C_3 and C_4 .

[0035] The segment C_1 has the real part 9, and the imaginary part runs from -7 to 7. The real part of $s(z)$ on curve C_1 is shown in Figure 3, and the imaginary part of $s(z)$ is depicted in Figure 4. The graphs for the remaining curve segments $C_{2,3,4}$ are similar.

[0036] As can be seen from the drawings, the calculation of the contour integrals is essentially a low-pass filtering.